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The Principle of Polyformity in Mathematics Education

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Abstract

The concept of mathematics is widely used in various societal spheres. As an educational subject, it engages students' logical thinking, contributing to the development of their competencies and thereby equipping them for specific life vocations and future career choices. Mathematics teaches us to be wise and to solve problems step by step. It is to be appreciated for being a logical puzzle, posing a challenge and fostering a readiness to engage with it in various ways. Anyone can learn mathematics; there is no human activity that does not depend on it. Almost all secondary schools and many universities include mathematics as a subject. The significance of the subject extends beyond education, playing a crucial role in shaping one's personality. The objectives and tasks of the educational subject are strictly defined, with primary education aiming at acquiring basic linguistic and mathematical literacy.

Keywords: mathematics, the principle of polyformity, polyformity as a didactic principle, motivation in mathematics education

Introduction. Mathematics and its significance

Mathematics is a dynamic field undergoing rapid development, reaching unprecedented scales. Its applications in the natural sciences and technology are vast, and the progress in these areas of human activity cannot be imagined without mathematics. The utilization of new mathematical disciplines in electronics and atomistics is already significantly transforming the world. The primary objective of modern technology is to replace human involvement in various physical and intellectual activities, improve living conditions, and unleash human energy for new, creative endeavors.

The concept of mathematics is widely used in various societal spheres. As an educational subject, it activates logical thinking in students, contributing to

the development of their competencies and preparing them for specific life vocations and future career choices. Mathematics teaches us to be wise and to solve problems step by step (Hilčenko, 2023). It is to be appreciated for being a logical puzzle, posing a challenge and fostering a readiness to engage with it in various ways. Anyone can learn mathematics; there is no human activity that does not depend on it. Almost all secondary schools and many universities include mathematics as a subject (Hilčenko, 2022).

The significance of the mathematics subject is immense, not only as an educational discipline but also in contributing to the development of one's personality. The objectives and tasks of the educational subject are strictly defined, with primary education aiming at acquiring basic linguistic and mathematical literacy. Pedagogical abilities of teachers and the classroom atmosphere are influenced by their personality traits and can be crucial for students' learning and the progression of their education. Teacher behavior shapes the image that students form about them, with the greatest impact occurring in the younger school age. The teacher's personality leaves a lasting impression on the students they teach. The role of a mathematics teacher is one of the most challenging and rewarding professions. Moreover, this profession carries a great deal of responsibility (Dragičević, 1994). Mathematics, as a subject, aims to acquire knowledge, skills, and habits. The process of acquiring knowledge within mathematics education is methodically elaborated, implying a quality and gradual path toward mastering the educational content.

The subject matter of the study. Objectives and tasks of mathematics education

Mathematics, as a subject, aims to acquire knowledge, skills, and habits, with the methodical development of knowledge acquisition within mathematics education signifying a quality and gradual path. The primary objectives of mathematics education in elementary school are to ensure that all students attain basic linguistic and mathematical literacy and participate in achieving appropriate educational achievement standards. Additionally, the goals include:

1. Equipping students to solve problems and tasks in new and unfamiliar situations (Hilčenko, Nikolić, 2022; Hilčenko, Jakovljević, Nikolić, 2021).
2. Enabling students to express and justify their opinions and engage in discussions with others.
3. Developing motivation for learning and interest in the subject matter.
4. Ensuring that students acquire elementary mathematical knowledge necessary for understanding phenomena and laws in nature and society.
5. Preparing students to apply acquired mathematical knowledge to solve diverse tasks in real-life situations.
6. Providing a foundation for successful continuation of mathematical education and self-education.

7. Developing mental abilities, forming a scientific worldview, and contributing to the well-rounded development of students' personalities (Dejić, 2008).

Mathematics Education:

1. Creating diverse opportunities for the realization of educational objectives, as well as the goals and tasks of mathematics education, through various contents and teaching methods.
2. Numerical literacy for successful engagement in any profession and achieving a quality life.
3. Acquiring knowledge essential for understanding quantitative and spatial relationships and laws in various phenomena in nature, society, and everyday life.
4. Gaining basic mathematical culture necessary for recognizing the role and application of mathematics in different areas of human activity (mathematical modeling), for successful continuation of education, and integration into the workforce.
5. Developing students' abilities in observation, perception, and logical, critical, analytical, and abstract thinking.
6. Cultivating cultural, work-related, ethical, and aesthetic habits in students, along with fostering mathematical curiosity.
7. Developing the ability to express oneself in mathematical language, ensuring clarity and precision in both written and oral communication.
8. Mastering fundamental facts about sets, relations, and mappings.
9. Mastery of basic operations with natural, whole, rational, and real numbers, along with understanding the basic properties of these operations.
10. Familiarity with essential geometric objects: lines, figures, and solids, and understanding their mutual relationships.
11. Training students for precision in measurement, drawing, and geometric constructions.
12. Preparing students to understand relevant content in natural and technical sciences.
13. Building positive characteristics in students' personalities, such as systematicity, persistence, accuracy, neatness, objectivity, self-control, and a sense of independent work.
14. Developing habits and skills in utilizing diverse sources of knowledge (http://oaji.net/articles/2016/1627-1453802551.pdf).

The teaching of mathematics, in addition to its educational role, also serves a formative purpose. Its significance lies in the development of intellectually robust individuals and the cultivation of work habits that are crucial for acquiring lasting active mathematical knowledge, serving as a foundation for the study of this subject at the next level of education. The aim is to enhance teaching methods that will enable students to independently acquire knowledge and apply it creatively.

Development (analysis of research results).

The principle of polymorphism

The didactic principle of polymorphism is not commonly encountered as a didactic specificity, and if it is applied in some instances, it is very rare, intuitive, spontaneous, singular, and incidental in the teaching of mathematics in primary and secondary schools, as well as at the university level. The essence of applying this principle lies in a continual emphasis on the integral consideration of various approaches to understanding and comprehending the studied teaching phenomena. Therefore, its exploitation in practice requires teachers to have an excellent understanding and skill in applying various pedagogical-didactic--methodological possibilities, inducing intensive cognitive activity in students expressed through quality self-disciplined work and increased motivation. The effectiveness of the polymorphism principle is based on the evident psychological fact that changes and diversity in work refresh the teaching process, while monotony generally induces a weakening of interest and the emergence of passivity and boredom (Marković, 2008). Due to its characteristics, the principle of polymorphism represents a universal teaching, scientific, and philosophical principle, with its epistemological foundation identical to that of the principles of permanence and the law of negation of negation, whereby the principle of polymorphism takes on the characteristic of a dialectical law. As all existing didactic principles are encompassed by the principle of polymorphism, it elevates this principle to the pedestal of universality (Nikolić, 2021). Đ.G. Marković, in his doctoral dissertation "New Perspectives on the Methodology of Teaching Mathematics in the Light of the Didactic Principle of Polymorphism" in 2007, introduced a new concept that combines the didactic principle of diversity and geometric polymorphisms. According to his understanding, the dominance of geometric polymorphisms and combined polymorphisms represents the principle of polymorphism, based on a finite number of conjunctions of logical laws or principles (laws of negation of negation, modus ponens, principles of obviousness, permanence, etc.). Based on the analysis of R. Arnheim's views on visual thinking and L. Vygotsky's ideas about the connection between thinking and speech, viewed in terms of the integrality of M. Marjanović's synthesis of the three-component nature of the concept, and based on personal thirty years of experience in direct teaching of high school mathematics and contemporary didactic tendencies based on dialectical laws, the author of the dissertation comes to the realization that the combination of verbal-textual and illustrative--demonstrative methods, viewed in the light of self-knowledge heuristics, provide unforeseen possibilities for the most effective "illumination" of teaching through the principle of polymorphism. This is especially noteworthy when it comes to the application of this principle in the form of content components of teaching, specifically the geometric polymorphism of the "shariginovsk type,"

which relates to various solutions to geometric problems using drawings and geometric representations of simple, as regular as possible, shapes such as triangles, squares, circles, etc. In this way, given problems are most simply represented, for example, by pictogrammatic notations, facilitating the easier formation of ideogrammatic representations of mental images, the so-called "aha noticing" of the three-component structure of the concept, types of polymorphism of the principle of obviousness. After all, the principle of polymorphism appears almost as an axiom, whose existence does not need to be separately proven (Nikolić, 2016).

Basic principles of polymorphism

The foundation of the principle of polymorphism, unlike the principle of permanence, lies in the dual or multiple application of the law of negation of negation to the same phenomena, i.e., initial problems or well-known theories. However, polymorphism is not merely a scientific and philosophical principle; it is also an educational principle. The essence of this significant educational principle is also reflected in the continuous emphasis on the integral consideration of various approaches to understanding and comprehending the studied educational phenomena (with the note that, whenever possible, geometric interpretations, i.e., schematizations of these phenomena, should be performed). The effectiveness of the principle of polymorphism is based on the fact that changes and diversity in work refresh the teaching process, while monotony generally induces a weakening of interest and the emergence of passivity and boredom. Due to these characteristics, the principle of polymorphism represents a scientific, didactic-methodological, and therefore philosophical principle, with its epistemological foundation identical to that of the principles of permanence and the law of negation of negation, whereby the principle of polymorphism acquires the characteristic of a dialectical law. As all existing didactic principles are encompassed by the principle of polymorphism, it elevates this principle to the pedestal of universality.

Polymorphism as a didactic principle

Polymorphism as a didactic principle is rarely mentioned in pedagogy, and if applied, it is often done intuitively, spontaneously, haphazardly, singularly, and incidentally in the teaching of mathematics in primary and secondary schools. It became evident very quickly that other forms of polymorphism, especially when combined with geometric and innovative content, have similar effects. This is clear when considering that geometric interpretations of arithmetic or algebraic problems, i.e., their representation through drawings, help students stabilize their internal representations. This is because visual thinking (thinking in images), due to its integral nature, produces "aha experiences" known to psychologists, triggered by a flash of complete clarity. This is evident as icons serve

as carriers of information, for which words are often unnecessary translations (Nikolić *et al.*, 2022). The need to classify polymorphic geometric interpretations, whose application in teaching creates an environment that inevitably induces activation and dynamism, in the didactics of mathematics prompted me to reconsider not only geometric but also arithmetic-algebraic polymorphism. It provided a comprehensive review of the entire methodology of mathematics teaching in the "light" of the didactic principles of polymorphism and permanence. Through several years of applying geometric and other polymorphisms in teaching practice, I have become convinced of the specific weight of geometric polymorphism in terms of intensity and importance. Therefore, the discovery of this methodological-didactic innovation is considered a product of both theoretical and empirical components. The practical application of didactic theory initiated the search for the best solutions to activate and dynamize the teaching process, permeating through empirical verification and recognizing polymorphism as a necessity in the educational process, hence the necessity for the complementation of didactic theory. This correlational relationship does not end here; it prompts the need for a theoretical treatment of the given phenomenon, i.e., a thorough theoretical analysis of this significant didactic principle.

Prim application of the principle of polymorphism in mathematics teaching

The essence of the didactic principle of polymorphism lies in the continual emphasis on the integral consideration of various approaches to understanding and comprehending the studied educational phenomena. Its exploitation in practice demands from teachers an excellent understanding and skill in applying various pedagogical-didactic-methodological possibilities, inducing intensive cognitive activity in students through high-quality self-disciplined work and increased motivation. Therefore, in mathematics teaching, the principle of polymorphism should play a universal role, presented by enriching the teaching with diverse content, tools, procedures, and methods (Nikolić, Hilčenko, 2023). When it comes to content, the focus is on selecting tasks that allow a greater variety of approaches to their solution and the use of obvious means. However, organizing such classes requires the adequate application of polymorphism in the forms and details of teaching methods, their variations, and even methodological innovations in the same lesson. The methodological forms and details that a teacher plans and applies during teaching are based on the timely pulsation of didactic principles, manifested in their simultaneous polymorphic-cohesive action, i.e., integral dialectical unity.

Polymorphism as a philosophical principle and direction in art

Polymorphism as a mode of thinking has always been a constant companion of renowned thinkers and artists worldwide. Examples of this include Archime-

des, Leonardo da Vinci, Isaac Newton, Nikola Tesla, and others. Polymorphic thinking significantly influenced their brilliant discoveries. Archimedes and Newton attempted and often succeeded in solving all the problems they tackled in multiple ways. Their constant search for different solutions, combined with a persistent emphasis on vivid geometric interpretations, sharpened their diamond--like minds and enabled epochal discoveries. Leonardo da Vinci, besides being a famous painter, anatomist, and architect, was also a leading mathematician of his time. The diversity of fields he engaged in, along with his iconic (visual) thinking expressed through geometric representations and sketches in polymorphic ways of the same phenomena, significantly contributed to all his findings (Marković, 2012). This is now well understood, especially with the results of experiments conducted by Rudolf Arnheim and other representatives of Gestalt psychology on visual thinking. Through drawings, we stabilize our internal representations because visual thinking, due to its integral nature, produces "aha experiences" known to psychologists, triggered by a flash of complete clarity. This is evident as icons serve as carriers of information, for which words are often unnecessary translations. Hence, it almost axiomatically follows why the accurate conclusion is that the same phenomenon studied and solved in a geometrically polymorphic way always triggers that "aha…" flash of complete clarity and integral, i.e., dynamic understanding of the essence of given problems. If we carefully observe the wondrous works of the divine creator – nature, we quickly discover that its most beautiful jewels, crystals, are symphonies of mathematics (Nikolić, 2016). Looking at the structures of snowflakes and other known crystals, the marvelous petals of various types of flowers, the shapes of shells, snail shells, sea stars, various trees and their leaves – in other words, looking anywhere around us, we will realize that nature's imagination in creating diverse forms of symmetry, bathed in a spectrum of incredible shades of colors, surpasses the imagination of even the greatest creators known in the history of art. Therefore, polymorphism is one of the essential natural characteristics or products of nature. Hence, it is not surprising that the universality of polymorphism, including polymorphism as a didactic principle, the phenomenon of polymorphic thinking, and polymorphism as a scientific and philosophical principle, as well as polymorphism as an artistic direction or style in art, is considered natural. Due to all of the above, it is entirely natural that mathematics takes pride in presenting polymorphism as a mode of thinking, polymorphism as a scientific and philosophical principle, and even polymorphism as an artistic direction or style in art.

Conclusions. Motivation in mathematics education

Mathematics, as an academic subject, plays a crucial role in fostering problem-solving abilities and logical reasoning skills among students. It is considered a challenging subject that demands continuous effort, time investment, and dedication. Many students may not always be willing to invest such effort, leading to difficulties in mastering mathematical concepts. However, when there is genuine interest in mathematics and learning occurs with enthusiasm, many obstacles dissipate, and the teaching and learning process unfolds more smoothly and successfully, with content absorption becoming more manageable.

In order to prevent school learning from being perceived as a "chore" or an activity to be completed solely to meet the expectations of teachers and parents, it is essential to motivate students to engage in their studies, i.e., to captivate their interest, awaken their will, and instill a love for the work, particularly in the context of learning mathematics. This can be achieved through special mathematical content, the beauty of its ideas and achievements, various methods, tools, and activities designed to make mathematics education enjoyable and to sustain that love over time. Various educational situations underscore the significant role of motivation in the learning process.

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