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Fostering Spatial Comprehension in Solid Geometry: The Role of Augmented Reality and GeoGebra

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Abstract

Augmented reality (AR) represents an innovative digital technology capable of fostering a learning environment that enhances the understanding of spatial concepts, mathematical objects, and the relationships between them. This paper looks into the features of AR and its practical applications in mathematics, highlighting its transformative potential in classrooms. Addressing challenges in solid geometry, such as the visualization and representation of three-dimensional (3D) spaces, AR emerges as a powerful tool, offering a dynamic means to explore and interact with geometric objects within real-world contexts.

A particular focus is placed on the *GeoGebra 3D Calculator* and its integrated AR module. Through four carefully selected examples from solid geometry, we provide detailed demonstrations of how these tools can be used to solve complex problems, thereby bridging the gap between theoretical concepts and practical understanding. These examples illustrate the profound impact of AR technology on improving spatial comprehension and engaging learners in mathematics.

Keywords: Augmented reality, GeoGebra, 3D Calculator, Solid geometry, Spatial visualization

Introduction

The study of solid geometry presents challenges due to the abstract nature of 3D spatial reasoning. Unlike planar geometry, which can be easily visualized, solid geometry requires learners to mentally construct and manipulate objects in space, a skill that is not intuitive for many students.

To address these challenges, educational technologies, especially AR, have been developed to create dynamic learning environments. AR offers a revolutionary method for visualizing geometric objects, making abstract concepts more tangible and comprehensible. Unlike static representations, AR integrates virtual objects into the physical environment, allowing learners to dynamically interact with them and view geometric elements from multiple perspectives, such as from the side, above, or below. This immersive approach fosters a deeper understanding of spatial relationships and geometric properties by enabling learners to explore these relationships in an intuitive, real-world context. (Bacca, Baldiris, Fabregat, Graf, 2014; Sekulic, Stojanov, 2023).

The *GeoGebra 3D Calculator*, with integrated AR functionality, provides a powerful platform for exploring solid geometry. It allows users to interact with 3D objects in augmented space, making abstract concepts more accessible (Widada et al., 2021). This paper examines the educational potential of AR in solid geometry, with a focus on the *GeoGebra 3D Calculator*, and demonstrates how AR can improve spatial comprehension through illustrative examples.

Teaching and Learning Solid Geometry in Computer Environment

Solid geometry presents challenges in teaching and learning, primarily due to the complexity of visualizing and representing 3D objects. Traditional methods relying on static diagrams often fail to fully convey the dynamic relationships of geometric structures. Computer technologies, like *GeoGebra* and AR, offer innovative solutions by enabling real-time interaction with 3D models in immersive environments (Petrov, Atanasova, 2020).

The *GeoGebra 3D Calculator* stands out for integrating AR, allowing learners to project geometric constructions into physical spaces and engage in hands-on exploration. Studies indicate that interactive tools enhance spatial reasoning and comprehension of solid geometry concepts (Sekulic et al., 2023). Additionally, these technologies support iterative learning and creative experimentation, reducing cognitive strain and enabling individual educational experiences.

The benefits of AR representations become particularly significant in the study of geometry, where understanding the interaction and spatial alignment of objects is crucial. By presenting objects as if they were part of the learner's real surroundings, AR bridges the gap between abstract mathematical constructs and their practical applications. For example, learners can manipulate virtual objects within their environment, observe their interactions, and analyze their spatial relationships from different angles. This interactive experience not only enhances engagement but also supports a more comprehensive understanding of mathematical principles, as noted in studies exploring AR's impact on education (Zulfiqar et al., 2023).

Despite the benefits, challenges like accessibility and technological proficiency remain barriers to adoption. Addressing these issues through professional development and limited access is crucial for broader implementation (Muhazir, Retnawati, 2020). Integrating computer technologies into solid geometry teaching transforms learning, bridging theoretical concepts with practical understanding.

GeoGebra 3D Calculator and AR Module

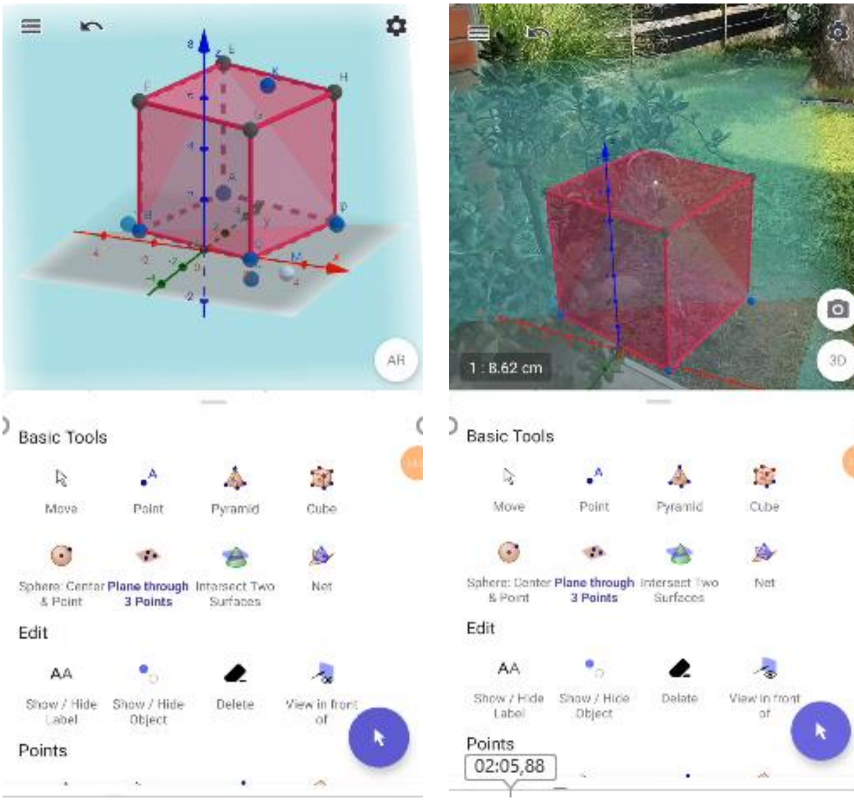
GeoGebra is a dynamic mathematics software designed for use in both mathematics and science education. As an open-source platform, it is freely available and offers a user-friendly interface, while still providing a powerful set of tools. *GeoGebra* includes several specialized modules, such as the *Graphing Calculator* (for plotting functions, exploring equations, and visualizing data), *Geometry* (for constructing figures, transformations, and measurements), *CAS Calculator* (for symbolic algebraic manipulations including solving equations, and computing derivatives and integrals), and the *3D Calculator* (for graphing and exploring three-dimensional objects and relationships). All these modules are accessible online or via download and are compatible with multiple platforms including Windows, iOS, Linux, and Android.

The *GeoGebra 3D Calculator* is particularly useful for teaching and learning topics in spatial geometry. It provides a wide range of tools for constructing 3D objects and conducting spatial investigations. Users can create planes (defined by three points, or parallel/perpendicular to a given object), intersect surfaces, and draw solids such as prisms, cones, cubes, and spheres. These tools are available to use within the *GeoGebra 3D Calculator* interface.

With the growing availability of mobile devices equipped with AR capabilities, AR has become increasingly accessible across platforms. For this reason, *GeoGebra's 3D Calculator with* AR module was selected for this study. It was chosen both for its cross-platform compatibility (e.g., Android and iOS) and because it is freely available. In this study, the Android platform was used, with a mobile phone serving as the device.

The *GeoGebra 3D Calculator* application for mobile phones can be easily downloaded from the *Play Store*. Once installed, it is ready for immediate use. The interface provides the familiar *GeoGebra* drawing space and standard *GeoGebra* tools, with the addition of buttons for switching between 3D and AR modes. When the 3D mode is selected, users can create and manipulate three-dimensional mathematical objects as usual, using *GeoGebra* tools, Figure 1a). However, selecting the AR mode integrates the virtual geometric content into the user's physical environment using the device's back camera, Figure 1b).

Once created, the object can be manipulated using a mobile phone, it can be observed from different angles, sides and positions in the environment chosen by the user. Precisely this possibility to manipulate and observe objects within the real environment is the key element which contributes to understanding spatial relations and to the learning process.



a) b)

Figure 1. 3D and AR representation of a cube in *GeoGebra 3D Calculator*

These features support dynamic, spatially contextualized learning experiences and illustrate the potential of integrating AR into mathematics education using accessible, free tools like *GeoGebra.*

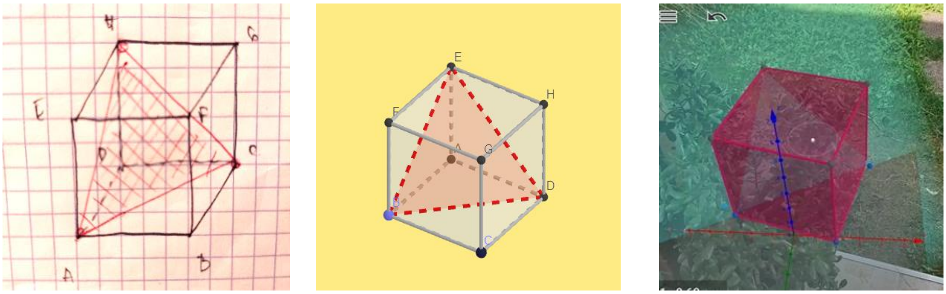
Application of the AR Module of the GeoGebra 3D Calculator for Solving Problems in Solid Geometry

By utilizing the AR module in the *GeoGebra 3D Calculator*, the four examples that demonstrate its diverse capabilities will be presented. A standout feature of *GeoGebra 3D Calculator* is its ability to project geometric problems into the real-world environment through AR modules, enabling students to directly observe and interact with these problems as though they were physically present.

Each of the four examples illustrates how the AR experience enhances understanding, allowing students to explore relationships among objects in a real setting, view them from different perspectives, and grasp spatial concepts more intuitively. The AR approach bridges theoretical knowledge and practical application, making it an indispensable tool for mastering spatial geometry.

> ***Problem 1:*** *The given object is a cube* $ABCDA_1B_1C_1D_1$ *with edges of length* a*. Calculate the area of the intersection between the cube and the plane defined by vertices* A, C, *and* D_1.
>
> ***Solution:*** $b = a\sqrt{2}$ *(diagonals of the faces of a cube),* ΔACD_1 *is equilateral, therefore:* $P = \frac{b^2\sqrt{3}}{4} = \frac{a^2\sqrt{3}}{2}$

Problem *1* is one classical geometry problem, which is often present within the school teaching and learning process. Considering the solving process of geometry problems, it is usual that the students are first instructed to make a sketch of the problem. This, for some students, can be an obstacle, because not all students are equipped with spatial reasoning and the ability to translate it on the twodimensional paper. Also, the graphical representation may not always be correct, and therefore can lead to wrong solving paths. By using *GeoGebra 3D Calculator*, from hand-drawn sketch (Figure 2a)), it can evolve into 3D representation (Figure 2b)), which can be further manipulated and viewed from different perspectives. Finally, using the AR module of the *GeoGebra 3D Calculator*, the problem can be exported in a real environment, and provided with even deeper insight, Figure 2c).



a) *b)* *c)*

Figure 2. Graphical, 3D and AR representation of Problem 1

The solving process enriched with the AR representation now becomes the process which transforms into more than just an application, it becomes an enriching experience or even an experiment, fostering deeper engagement and exploration, Figure 3. As a result, this approach not only enhances understanding but also leads to an effective solution of the problem at hand.

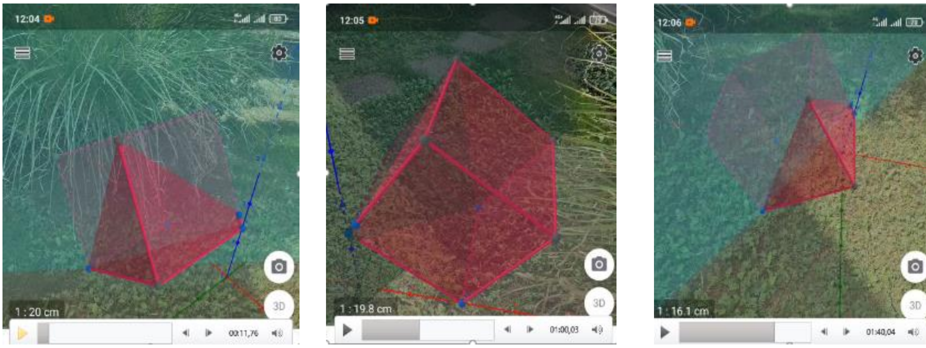


Figure 3. AR illustration of the solving process of Problem 1

Problem 2: *A cube with edge length* a *is given. Calculate the distance from one vertex to a spatial diagonal that does not pass through that vertex.*

Solution: *Let the desired distance be* d*, and let* x *and* y *be the segments into which the foot of the perpendicular divides the diagonal. From this:* $x + y = a\sqrt{3}$*,* $d^2 + x^2 = a^2$*,* $d^2 + y^2 = 2a^2$*.The desired distance can then be easily find as:* $d = \frac{a\sqrt{2}}{\sqrt{3}}$.

In Problem *2* the option of first creating a 3D representation of the problem is illustrated, and within it, the use of the possibility to extract some features which are an important part of the solution. This is especially important for this problem, considering that spatial diagonal is one part of it, and therefore, the hand-drawn sketch can be very challenging even for students having good spatial perception.

The *GeoGebra 3D Calculator* even offers this kind of option to switch on/off some parts of the 3D model, in order to enable better insight into the problem. Further use of the AR module places the created object into the real environment and by that offer deeper experience, Figure 4.

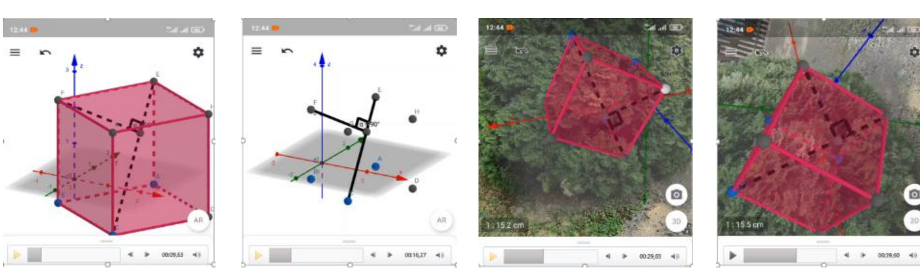


Figure 4. 3D and AR representation of Problem 2

Problem 3: *The edge of a regular tetrahedron has length* a. *Calculate the area of the cross-section of the tetrahedron with a plane that contains one edge of the tetrahedron and divides the opposite edge in a* $2:1$ *ratio.*

Solution: *Observe that the intersection of the tetrahedron and the given plane is an isosceles triangle with side length b and height h. By applying the law of cosine, we find that:* $b = \frac{a\sqrt{7}}{3}$, *and* $h = \frac{a\sqrt{19}}{6}$. *The area of the desired triangle is:* $P = \frac{a \cdot h}{2} = \frac{a^2\sqrt{19}}{12}$.

Similar to previously considered problem, Problem *3* requires insight into the internal structure, this time of the tetrahedron. Figure 5 illustrates the application of the 3D and AR representation of the tetrahedron.

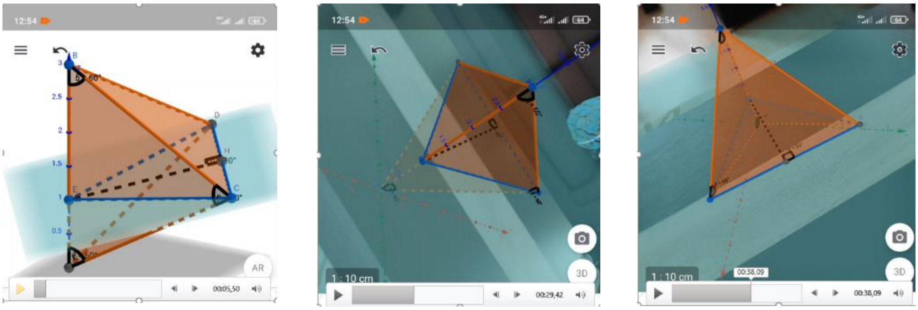


Figure 5. 3D and AR representation of Problem 3

Problem 4: *A sphere is circumscribed around a right circular cone whose slant height is equal to the diameter of the base. What is the ratio of the surface areas of the cone and the sphere?*

Solution: *We can observe that the radius of the sphere R is equal to* $\frac{2}{3}$ *of the height of the cone H.*

From this it follows: $\frac{P_K}{P_L} = \frac{\frac{a^2 \cdot \pi}{4} + \frac{a \cdot \pi \cdot a}{2}}{4\pi \cdot \frac{a^2 \cdot 3}{9}} = \frac{9}{16}$.

The cone circumscribed into the sphere, as presented in Problem *4*, refers to a group of geometry problems which students find the most challenging. The possibility to realistically represent these kinds of objects is of great importance for students, because they can gain insight into the relationships between two or more geometric objects. Particularly advantageous is the capability offered by *GeoGebra 3D Calculator* to isolate or exclude parts of the objects, as this allows for observing and determining the relationships among the objects, leading to the solution of the problem through further analysis, Figure 6.

Figure 4. 3D and AR representation of Problem 4

Conclusions

The integration of AR into mathematics education, particularly through tools like the *GeoGebra 3D Calculator*, has the potential to significantly transform the teaching and learning of solid geometry. As demonstrated through the presented examples, AR technology enhances students' ability to visualize, interact with, and understand complex 3D objects and their relationships in space. By bridging the gap between abstract mathematical concepts and their real-life representations, AR fosters a more intuitive and engaging learning environment.

Traditional methods of teaching solid geometry often rely on two-dimensional sketches and static diagrams, which can be limiting for students who struggle with spatial reasoning. The *GeoGebra 3D Calculator* addresses this challenge by enabling dynamic construction, manipulation, and visualization of geometric figures. Furthermore, the AR module extends these capabilities by embedding these figures into the learner's physical environment, making the exploration of geometric properties a more meaningful experience.

The examples presented highlight how AR can enrich the problem-solving process by moving from conceptual sketches to interactive 3D models and finally to contextualized AR experiences. This transformation not only aids comprehension but also encourages experimentation, active learning, and deeper engagement with mathematical content.

In conclusion, AR enhanced learning experiences in solid geometry represent a powerful innovation. By making spatial relationships visible and manipulative within real-world contexts, AR helps students develop critical mathematical competencies and fosters a deeper appreciation for the structure and beauty of geometry. These findings highlight AR's potential as a powerful educational tool, capable of transforming how mathematical and spatial concepts are taught and understood (Hilčenko, Nikolić, 2023, 2024). Incorporating AR technology into mathematical education aligns with the evolving needs of modern learners, providing innovative ways to make learning engaging and effective.

References

Bacca, J., Baldiris, S., Fabregat, R., Graf, S. (2014). Augmented Reality Trends in Education: A Systematic Review of Research and Applications. *Journal of Educational Technology & Society*, *17*(4), 133–149. https://doi.org/10.1109/access.2023.3331218.

Hilčenko, S., Nikolić, S. (2023). CHILD: "I don't understand – we didn't learn that in kindergarten!" *Journal of Education, Technology and Computer Science*, *4*(34), 41–48.

Hilčenko, S., Nikolić, S. (2024). I'm a retiree my brain after the age of 60. *Journal of Education, Technology and Computer Science*, *5*(35), 127–131.

Muhazir, A., Retnawati, H. (2020). The teachers' obstacles in implementing technology in mathematics learning classes in the digital era. *Journal of Physics Conference Series*, *1511*(1), 012022. https://doi.org/10.1088/1742-6596/1511/1/012022.

Petrov, P., Atanasova, T. (2020). *Developing Spatial Mathematical Skills Through Augmented Reality and GeoGebra*. ICERI Proceedings.

Sekulić, T., Stojanov, J. (2023). Augmented Reality Learning Environment for Mathematics and Sciences in GeoGebra 3D. In: *Proceedings of the XIV International Conference of Information Technology and Development of Education – ITRO 2023* (pp. 61 – 65). University of Novi Sad, Technical Faculty „Mihajlo Pupin" Zrenjanin, Republic of Serbia.

Sekulić, T., Manigoda, G., Kostić, V. (2023). Application of the 3D Geogebra Calculator for Teaching and Learning Stereometry. In: *Proceedings of Sinteza 2023 – International Scientific Conference on Information Technology and Data Related Research* (pp. 166–171). Belgrade: Singidunum University, Serbia.

Widada, W., Herawaty, D., Nugroho, K.U.Z., Anggoro, A.F.D. (2021). Augmented Reality assisted by GeoGebra 3-D for geometry learning. *Journal of Physics Conference Series*, *1731*(1), 012034. https://doi.org/10.1088/1742-6596/1731/1/012034.

Zulfiqar, F., Raza, R., Khan, M.O., Arif, M., Alvi, A., Alam, T. (2023). Augmented Reality and its Applications in Education: A Systematic Survey. *IEEE Access*, *11*, 143250–143271.