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Learning Geometric Polyformisms in Mathematics Education

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Abstract

The renowned Russian mathematician, mathematics education methodologist, scientist, science popularizer, author of geometry textbooks, and lecturer at Moscow State University, Igor F. Sharygin, believed that geometry should primarily be *geometric*, rather than analytical or algebraic. The central character in this story should be the *figure*, with the triangle and circle dominating its surface, and the main means of learning should be the drawing and the image. Textbooks that focus on geometric content should not be limited to the development of geometric theories. The learning process of such content involves a wide variety of work formats, primarily through problem solving. A problem is not merely a skill exercise, but a component of knowledge. Students should become familiar with a sequence of sufficiently challenging geometric problems, following well-known models. Incidentally, this essentially constitutes the process of learning algebra as well.

Keywords: geometry, polyformism, geometric polyformisms in teaching

Introduction

We present students with methods and convey algorithms that are difficult, if not impossible, to discover independently. In geometry, unlike algebra, such algorithms are scarce or almost non-existent. Nearly every geometric problem is non-standard. Therefore, in teaching, the importance of key problems increases – those that explain useful facts or illustrate a method (Sharygin, 2004). Drawing is the first step toward abstraction – essential properties are condensed, and non-essential ones are disregarded (Sharygin, 2004). When Rudolf Arnheim, one of the founders of the Gestalt school in psychology, wrote his seminal work *Visual Thinking* (the unity of image and concept) in the early 1930s, he based all his claims on geometric interpretations. In his article "Does Geometry Belong in 21st Century Schools?", Igor F. Sharygin emphasizes that we create geometric images in order to stabilize our internal representations. Visual thinking – thinking in images – has the property of comprehensiveness and is not easily transferable. Images, or icons, are carriers of information. That is why Sharygin (1937–2004), when speaking about "good geometry," puts a good problem – presented with a beautiful image and vivid language – at the center of the story. This "vivid language" makes visual thinking more transferable. The interpretation of a mathematical problem through geometric polyformism allows for a dynamic approach to the problem or phenomenon itself, resulting in comprehensive and profound understanding (Nikolić, 2021; Hilčenko, Nikolić, 2023, 2024).

When we say that mathematics teaching should be dominated by geometric polyformisms, we refer to instruction where mathematical problems are primarily solved and teaching phenomena are explained through various schematic representations–that is, through geometric reinterpretations of the same problem in multiple ways (Nikolić, Hilčenko, 2024).

Polyformism

The fundamental principles of polyformism are based on the dual or multiple applications of the law of the negation of the negation to the same phenomena–i.e., to initial problems or established theories. The interpretation of a mathematical problem that allows for polyformal geometric analysis enables a dynamic approach to the problem or the phenomenon itself, resulting in its comprehensive and profound understanding. The diversity dominated by geometric polyformisms represents the principle of polyformism, which is grounded in a finite number of logical conjunctions or principles (e.g., the laws of the negation of the negation, modus ponens, the principles of obviousness, permanence, etc.) (Marković, 2012). This diversity, when combined with arithmetic, algebraic, and methodological variation, constitutes a didactic principle of polyformism. At its core, this principle lies in the constant insistence on an integrative view of various evident – especially geometric – approaches to the understanding and conceptualization of taught mathematical notions (Nikolić, Hilčenko, 2024). In practice, this demands that the teacher possesses a deep knowledge of and the ability to apply a wide array of professional, didactic, and methodological strategies. At the same time, it stimulates students' intensive intellectual activity, expressed through high-quality, self-directed work and enhanced motivation. Instruction, when viewed through the lens of such principled foundations, presupposes new, polyformal methodological approaches. Learning through self-cognitive polyformal heuristics – as a dominant method within the framework of polyformal principles of interactive teaching – implies that the content to be acquired by students is not presented in a ready-made form, but must instead be discovered, preferably in multiple ways (Nikolić, Đokić, Hilčenko, 2022). This significantly enhances students' intellectual capacity, motivation, and learning engagement, accompanied by a sense of satisfaction from the accomplished work. Learning through the method of self-cognitive polyformal heuristics yields greater outcomes in terms of acquiring conceptual knowledge, and especially procedural (i.e., applicable) knowledge, in accordance with modern taxonomies of knowledge. This occurs because the student invests individual effort to organize newly acquired information within their own cognitive system and to find the full range of necessary information. As a result, the student's ability to organize and structure data improves, through deductive and analytical-synthetic approaches and their application to various problem-solving and even real-life contexts. According to numerous researchers, modern education – which represents a fusion of principled and methodological "weaving," aided by the use of computers (often unrecognized or unacknowledged by traditionalist approaches) – introduces new qualities of diverse instructional practices. These enhance student engagement in the learning process, increase motivation, curiosity, initiative, creativity, and the applicability of acquired knowledge in everyday life, which are the core goals of contemporary mathematics education (Nikolić, 2021).

The Didactic Principle of Polyformism

The effectiveness of the polyformism principle is based on an evident psychological fact: change and variety in instructional work refresh the teaching process, whereas monotony typically induces a decline in interest and results in passivity and boredom. Therefore, in mathematics education, the principle of polyformism should play a general role – one that is manifested through the enrichment of instruction by means of diverse content, tools, procedures, and methods. With regard to content, this refers to the selection of tasks that allow for multiple, varied approaches to problem-solving, including the use of visual and concrete teaching aids. However, organizing such lessons requires the appropriate application of diverse methodological forms and instructional variations within a single lesson. The methodological forms and specific teaching strategies planned and implemented by the teacher during instruction are grounded in the timely activation of didactic principles. This manifests as their simultaneous *polyformal-cohesive effect* – that is, their integral dialectical unity (Nikolić, Marković, 2016).

Development (Analysis of Research Results): Examples of Geometric Polyformism in Primary School Teaching

In primary and secondary school textbooks, as well as in various problem collections and mathematical handbooks, there is typically only one, or at most two to three, approaches to deriving a given formula. These proofs are generally based on theorems concerning decomposable or complementary equality of polygonal areas.

Proof 1

Let ABCD be a trapezoid with bases AB and CD of lengths a and b, respectively, and height h. By drawing the diagonal AC, the trapezoid is divided into two triangles: △ABC and △ACD The area of the trapezoid can thus be calculated as the sum of the areas of these two triangles, using the aforementioned theorem on decomposable equality of polygonal areas:

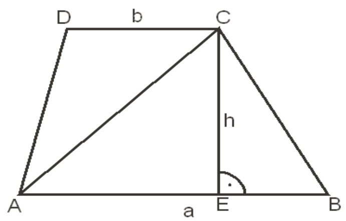


Figure 1

$$P_{ABCD} = P_{\Delta ACD} + P_{\Delta ABC} = \frac{a \cdot h}{2} + \frac{b \cdot h}{2} = \frac{(a+b) \cdot h}{2},$$

which is the required result (see Figure 1).

Proof 2

Let GF be the midline (median) of trapezoid. Let point E denote the intersection of lines DF and AB, as shown in Figure 2. It is easy to demonstrate the congruence of triangles △DGF and △AFE based on the well-known SAS (side–angle–side) triangle congruence criterion. Consequently, their areas are equal:

$$P_{\Delta DFC} = P_{\Delta EFB}.$$

According to the theorems on decomposable and complementary equality of polygonal areas, the area of trapezoid ABCD is equal to the area of triangle △AED, i.e.,

$$P_{ABCD} = P_{ABFD} + P_{\Delta DFC} = P_{ABFD} + P_{\Delta EFB} = P_{\Delta AED} = \frac{(a+b) \cdot h}{2},$$

Which was to be demonstrated.

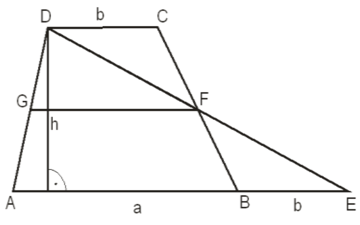


Figure 2

Proof 3

Let the measures of the lengths of the bases of the trapezoid AB and DC be a and b, respectively, where, as in the previous cases, a > b, see Figure 3. Through vertex C, construct a line CE||DA. It is easy to observe that the length of segment AE = b, as is EB = a – b.

The area of the trapezoid can then be decomposed into the sum of the area of the parallelogram AECD and the area of triangle △EBC, i.e.,

$$P_{ABCD} = P_{AECD} + P_{\Delta EBC} = b \cdot h + \frac{(a-b) \cdot h}{2} = \frac{(a+b) \cdot h}{2}.$$

During supplementary mathematics classes for upper elementary school students, we assigned a task in which the students were encouraged to independently discover additional algorithms for deriving the formula for the area of a trapezoid. We instructed them that they could use the stated theorems on decomposable and supplementary equality of polygonal areas, as well as other geometric principles – such as congruence, homothety, and similarity of geometric figures. With the help of semi-guided and independent heuristic approaches, the students arrived at the following polyform procedures.

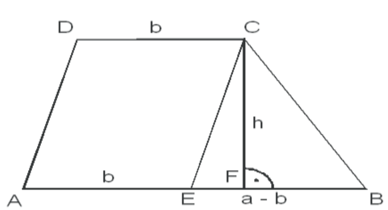


Figure 3

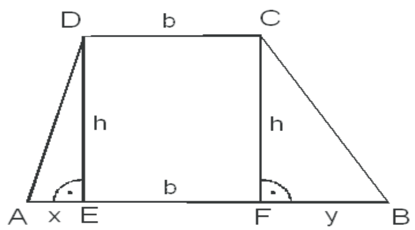
Proof 4

If we modify Figure 3 by removing the line segment CE and instead construct a new line CE perpendicular to AB, we obtain Figure 4. In this configuration, we have:

DE = CF = x, AE = x, FB = y, EF = b, AB = x + b + y = a.

The area of trapezoid ABCD can be decomposed into the sum of the areas of triangles △AED, △FBC, and the rectangle EFCD, that is:

$$P_{ABCD} = P_{\Delta AED} + P_{AFCD} + P_{\Delta FBC} = \frac{x \cdot h}{2} + b \cdot h + \frac{y \cdot h}{2} = $$
$$= \frac{(x + b + y) \cdot h}{2} + \frac{b \cdot h}{2} = \frac{(a + b) \cdot h}{2}$$



Slika 17d

Figure 4

Proof 5

Let FG be the midline of the trapezoid, and let the line FL be parallel to AD, such that point **E** is the intersection of lines AB and FL. It is easy to observe that the area of trapezoid ABCD is equal to the area of parallelogram AELD, which, based on the theorem of equidecomposability of polygonal areas, can be expressed as the sum of the areas of the pentagon AEFCD and triangle △FLC (see Figure 5), that is:

$$P_{ABCD} = P_{AEFCD} + P_{\Delta EBF} = P_{AEFCD} + P_{\Delta LCF} = P_{AELD} = \frac{(a + b) \cdot h}{2}$$

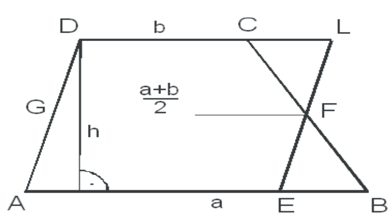


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Figure 5

since, by the SAS congruence criterion (side–angle–aide), the triangles are congruent. $\Delta EBF \cong \Delta LCF$. Now, let us examine two dynamic proofs.

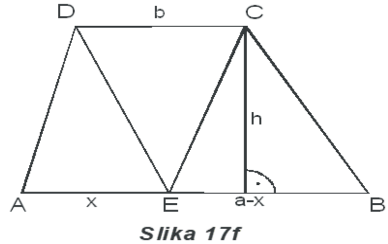

Figure 6

Proof 6

Let us choose an arbitrary point **E** on the base **AB** of trapezoid **ABCD**, and construct the segments accordingly (see Figure 6). Let **h** denote the height of the trapezoid. It is easy to observe that, based on the theorem of equidecomposability of polygonal areas, the area of trapezoid **ABCD** is equal to the sum of the areas of triangles △AED, △DCE and △EBC that is:

$$P_{ABCD} = P_{\Delta AED} + P_{\Delta DCE} + P_{\Delta EBC} = \frac{x \cdot h}{2} + \frac{b \cdot h}{2} + \frac{(a-x) \cdot h}{2} = \frac{(a+b) \cdot h}{2}.$$

Proof 7

If point E is chosen on the line AB such that the order of points is either A–B–E or E–A–B, two similar proofs can be constructed. Here is one of them, corresponding to the case where the order is A–B–E, as shown in Figure 7.

The area of the larger trapezoid **AECD** can be calculated in two different ways:

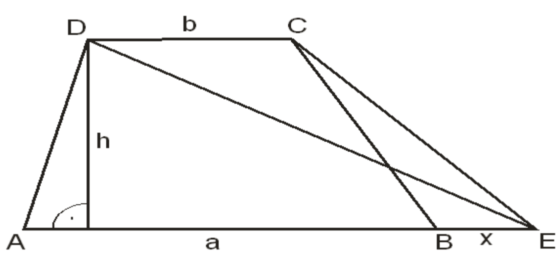

Figure 7

$$(1) \quad P_{AECD} = P_{ABCD} + P_{\Delta BEC} \text{ and}$$

$$(2) \quad P_{AECD} = P_{\Delta AED} + P_{\Delta DCE},$$

from equations (1) and (2), it follows that

$$P_{ABCD} + P_{\Delta BEC} = P_{\Delta AED} + P_{\Delta DCE} \text{, i.e.}$$

$$P_{ABCD} = \frac{(a+x)\cdot h}{2} + \frac{b\cdot h}{2} - \frac{x\cdot h}{2} = \frac{(a+b)\cdot h}{2}.$$

Proof 8

Through vertex B in the plane of trapezoid ABCD, construct a line parallel to AD, i.e., let BE be such that BE||AD. This line intersects the line containing the base **DC** at point **E** (see Figure 8).

The area of parallelogram **ABED** can be decomposed into the sum of the areas of trapezoid **ABCD** and triangle △ECB, that is: Area

$$P_{ABED} = P_{ABCD} + P_{\Delta ECB}$$

from which it follows:

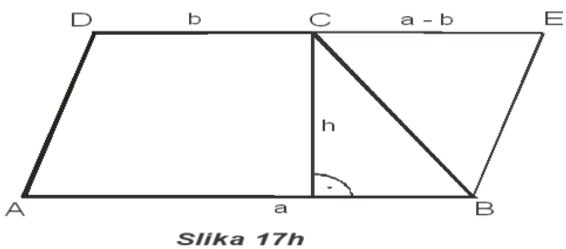

Figure 8

$$P_{ABCD} = P_{ABED} - P_{\Delta ECB} = a\cdot h - \frac{(a-b)\cdot h}{2} = \frac{(a+b)\cdot h}{2}$$

which was to be proven.

Proof 9

Here is another proof similar to Proof 2 (see Figure 9).

Let trapezoid ABCD be given. Construct its midline GF and the segments DF and AF, where AB = a and DC = b are the bases, and h is the height of the trapezoid. Using the theorem of equidecomposability of polygonal areas, the area of the trapezoid can be expressed as the sum of the areas of triangles △ABF, △GFA, △GFD and △DCF, that is:

$$\Rightarrow P_{ABCD} = \frac{a\cdot\frac{h}{2}}{2} + 2\cdot\frac{\frac{a+b}{2}\cdot\frac{h}{2}}{2} + \frac{b\cdot\frac{h}{2}}{2} = \frac{(a+b)\cdot h}{2},$$

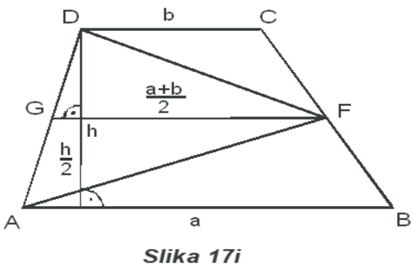

Figure 9

which completes the proof.

Proof 10

Now, here is an interesting proof in which we calculate the area of a parallelogram that is twice the area of the required trapezoid. Construct trapezoid CFEB congruent to trapezoid ABCD, such that.

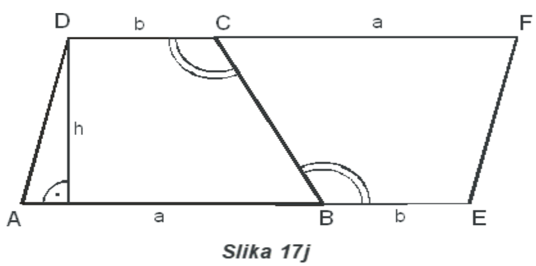

Figure 10

Which is obtained by rotating trapezoid ABCD around the midpoint of leg BC by 180 degrees, as shown in Figure 10. It is easy to observe that the area of parallelogram ADEF is decomposable into the sum of the areas of the congruent trapezoids, that is: $P_{AEFD} = 2 \cdot P_{ABCD}$ therefore, it is

$$P_{ABCD} = \frac{P_{AEFD}}{2} = \frac{(a+b) \cdot h}{2}$$

which was to be proven.

Proof 11

This derivation, in addition to previously exploited theorems, is also based on the application of the properties of homothety, i.e., similarity. Let S denote the intersection of the lines containing the legs AD and BC of trapezoid ABCD, and let SE = h_1 denote the height of triangle △DCS, and the height of the trapezoid

EF = h, with AB = a and DC = b being its bases, where a > b. Then, from the similarity of triangles (see Figure 11), it follows that:

$$\frac{h_1}{b} = \frac{h_1 + h}{a} \Rightarrow h_1 = \frac{h \cdot b}{a - b},$$

therefore, it is easy to observe the following relation:

$$P_{ABCD} = P_{\Delta ABS} - P_{\Delta DCS} = \frac{(h_1 + h) \cdot a}{2} + \frac{h_1 \cdot b}{2} = \frac{\left(\frac{h \cdot b}{a - b} + h\right) \cdot a}{2} - \frac{\frac{h \cdot b}{a - b} \cdot b}{2} =$$

$$= \frac{h}{2} \cdot \frac{b \cdot a + a^2 - a \cdot b - b^2}{a - b} = \frac{h \cdot (a^2 - b^2)}{a - b} = \frac{(a + b) \cdot h}{2}$$

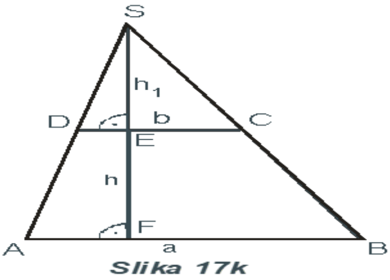

Figure 11

which completes the proof. Such polymorphic approaches to solving geometric problems enrich mathematics education, inducing greater activity and dynamism in students' work, as well as a more complete understanding of the given problems.

Advantages of Learning Through the Method of Self-Discovery Polyform Heuristics

The diversity dominated by geometric polyformism, in combination with arithmetic, algebraic, and methodological variability, represents a didactic principle of polyformity. This principle is grounded in a finite number of logical conjunctions or laws and principles (such as the law of double negation, modus ponens, the principles of obviousness, permanence, etc.).

The essence of this important instructional principle lies in a permanent insistence on an integral consideration of diverse, especially geometric, approaches to understanding and conceptualizing educational content. In practice, this requires teachers to possess thorough knowledge and skill in applying a wide range of professional, didactic, and methodological strategies. It also stimulates intensive cognitive engagement among students, expressed through high-quality, self-directed work and increased motivation.

Instruction, when viewed through the lens of these principled foundations, necessarily entails new polyform methodological approaches. Within such an interactive teaching model, the method of self-discovery polyform heuristics becomes dominant. Here, the learning content is not presented in its final form; instead, it must be uncovered – preferably through various pathways (Nikolić, 2022). This process enhances students' intellectual capacities, motivation, and engagement, and it fosters a sense of satisfaction through accomplishment. Learning through the self-discovery polyform heuristic method produces stronger effects in acquiring not only substantive knowledge but especially procedural and applicable knowledge, as outlined in modern taxonomies of learning. In this process, students invest their own effort to organize newly acquired information within their personal cognitive frameworks and to seek out the entire spectrum of necessary knowledge. This, in turn, improves their ability to structure and manage data using deductive, analytic-synthetic approaches and to apply such methods in solving various academic and real-life problems. According to numerous researchers, modern teaching – conceived as a synthesis of principled and methodological "weaving", often supported by computer technology and unrecognized by traditionalist pedagogies – offers new qualities in diverse teaching practices. It enhances student engagement, improves knowledge acquisition, and fosters greater motivation, curiosity, initiative, creativity, and applicability of acquired knowledge in everyday life. These are among the fundamental goals of contemporary mathematics education (Marković, Veljić, 2015). Although such research is still rare in our region, it is increasingly relevant worldwide. The self-discovery heuristic method is precisely the approach that modern education needs – one that the school of the 21st century is bound to "discover" and affirm. We are convinced that, through its practical revelations and educational "resurrections", it will ultimately earn the status of universality.

Conclusion

The essence of this important didactic principle lies in the continual emphasis on an integrated view of diverse approaches to understanding and conceptualizing educational phenomena. Its application in practice requires teachers to possess a high level of expertise and skill in employing a wide range of professional, didactic, and methodological strategies. At the same time, it stimulates intensive cognitive engagement from students, expressed through quality self-directed effort and increased motivation. The effectiveness of the principle of polyformity is grounded in a well-established psychological fact: variation and change in instructional practice refresh the learning process, while monotony typically leads to diminished interest, passivity, and boredom. For this reason, the principle of polyformity should play a universal role in mathematics education – enriching the learning process through diverse content, tools, techniques, and methods. Due to these characteristics, the principle of polyformity represents not only a didactic--methodological principle but also one whose epistemological foundation aligns with that of the principle of permanence and the law of the negation of negation. In this way, the principle of polyformity assumes the features of a dialectical law. As the principle of polyformity encompasses all existing didactic principles, it is elevated to the status of a universal principle in education.

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