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New Dynamic Method for Examination of Elastic Properties of Thin Wire Samples

Introduction

Elastic modulus (also called *tensile modulus* or *Young modulus*) *E* belongs to the most important material constants. It determines the relation between stress σ along the axis, and strain ε at axial loading, in the form of

$$\sigma = E\varepsilon ,$$

which is valid in the range of Hooke's law. Higher loading of the sample may result in exceeding the limits of elastic behaviour of the material.

There are several possibilities how to measure this quantity. The best know methods are as follows: mechanical (static and dynamic), acoustic, ultrasonic, resonant, optical, etc. Mechanical methods are the most suitable for measuring elastic modulus *E* of thin samples, such as rods, wires, columns, fibres, etc. Application of the static methods (e.g. direct prolongation, two- and three- point bending etc.) however, is rather disadvantageous, as they can hardly reach accuracy better than 10% [Brown 1969].

Higher accuracy can be reached by means of dynamic methods. Elastic modulus *E* can be determined with several percent accuracy by means of vibrating samples at two- or three-point bending [Tomoshenko, Young, Weawer 1974], or by balance of so called Searl´s pendulum [Agrawal, Jaim, Sharma 2008]. This paper presents a new dynamic method – the method of reverse pendulums connected by a measured wire sample. Modulus of the wire elasticity can be calculated after measuring constrained parameters of the vibrating system.

Measuring equipment

A diagram of used equipment is shown in Fig. 1. Both reverse pendulums were hung so that they vibrated in a common plane. When using a classical spring connection for demonstration of composition of parallel oscillations, we can determine the spring's stiffness, too.

In our experiment an elastic wire shaped like a horizontal circle was used as a connection. Deviation of the pendulums in their common plane gave rise to bending vibrations of the wire, while the same phenomenon as in the case of spring connection (i.e. energy transfer from one pendulum to another, formation of impacts, etc.) could be observed.

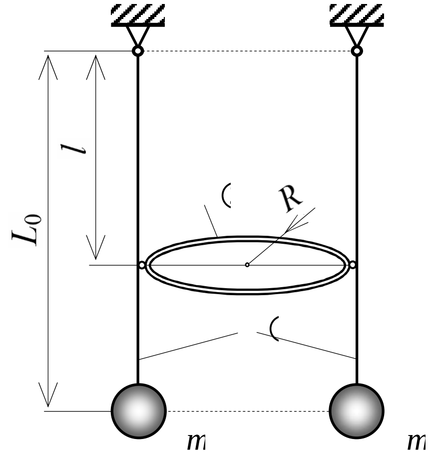
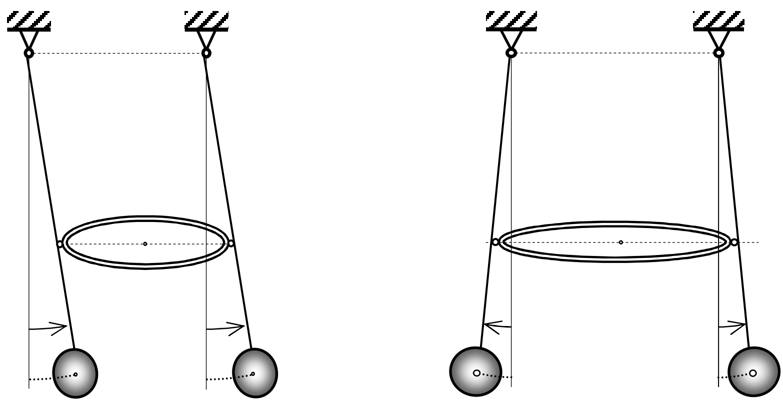


Fig. 1. Measuring equipment scheme
(1 – pendulums, 2 – wire sample connecting pendulums)

Young modulus of the wire elasticity can be determined similarly as the spring stiffness can be specified. Corresponding basic circular frequencies ω_1 and ω_2 necessary for calculation can be determined in two ways: either by means of impacts (detailed description will be noted later) or by experiments shown in Fig. 2a and 2b examining concordant and/or discordant oscillations of the pendulums.



a) b)

Fig. 2. Vibrational modes of coupled pendulums
a) 1st mode – concordant vibrations, b) 2nd – discordant vibrations

Dynamic Analysis

When determining elastic modulus E from our resultss, it is necessary to specify the range of the wire circular are deformation caused by the force $\boldsymbol{F}$ (Fig. 3a).

($\boldsymbol{G}$ is weight of pendulum, u is a wire deformation; L_0 is distance of the pendulum centre from the rotation axis, φ is angle of the pendulum deviation and l is distance of the wire connection from the pendulum point).

To do so we used strain energy A the quantity of which is given by bending effects in particular. Regarding perpendicular axes symmetry, the calculation was done only for a quadrant (Fig. 4).

Strain energy A of the quadrant is as follows:

$$A = \frac{1}{2EJ_z} \int_0^{\frac{\pi}{2}} M^2(\psi) R \mathrm{d}\psi , \tag{1}$$

where

$$M(\psi) = M_0 - \frac{F}{2} R(1 - \cos\psi) \tag{2}$$

and E is elastic modulus, J_z is area moment of inertia about wire neutral axis, $M(\psi)$ is bending moment, R is arc radius and ψ represents the angle of turning of the arc. The values of T and N correspond to tangential and normal component of the force F.

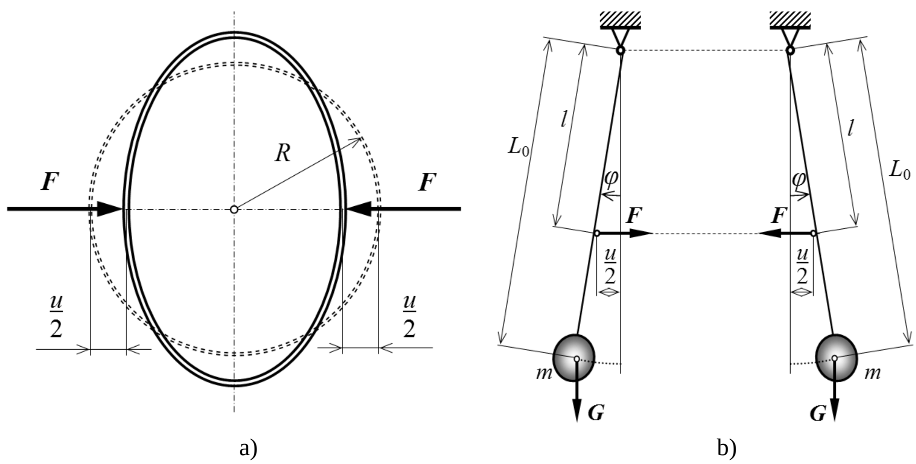


Fig. 3. Deformation diagram at transfer of force *F*: a) to the circular wire, b) to the pendulum

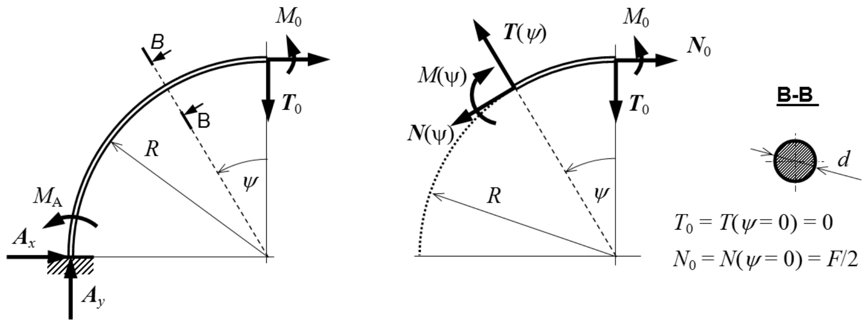


Fig. 4. The analysis of internal forces during deformation of circular wire

Before calculations it is necessary to determine the value of bending moment M_0, which corresponds to zero rotation at the point $\psi = 0$, i.e.

$$\frac{\partial A}{\partial M_0} = \frac{1}{EJ_z}\int_0^{\frac{\pi}{2}}\left[M_0 - \frac{F}{2}R(1-\cos\psi)\right]R\mathrm{d}\psi = 0\,, \tag{3}$$

$$M_0 = \frac{F}{2}R\frac{(\pi-2)}{\pi}\,. \tag{4}$$

The value of displacement u_1 at the point $\psi = 0$ can be determined from the following condition:

$$u_1 = \frac{\partial A}{\partial N_0} = \frac{\partial}{\partial N_0}\left[\frac{1}{2EJ_z}\int_0^{\frac{\pi}{2}}\left(M_0 - N_0R(1-\cos\psi)\right)^2 R\mathrm{d}\psi\right]. \tag{5}$$

Total displacement u is the given by the equation:

$$u = 2u_1 = \frac{FR^3}{4EJ_z}\left(\frac{\pi^2-8}{\pi}\right). \tag{6}$$

Elastic modulus can be determined also from frequencies ω_1 and ω_2 of the connected pendulums. If the interaction between connecting circle element and pendulums is replaced by its force effect then moment M applied on the pendulum (Fig. 3b) can be determined as

$$M = mgL_0\sin\varphi + Fl\,, \tag{7}$$

where m is pendulum weight and g is gravity acceleration.

Supposing that the pendulums are oscillating in the field of small oscillations ($\varphi < 5°$), $\sin\varphi \cong \varphi$ and $u = 2l\varphi$. Thus, using expressions (6) and (7) we can obtain a new relation:

$$M = mgL_0\varphi + \frac{8\pi l^2 EJ_z}{R^3(\pi^2 - 8)}\varphi . \quad (8)$$

Setting this relation into motion equation of the pendulum we can calculate circular frequency for discordant oscillations of the connected pendulums:

$$\omega_2^2 = \frac{1}{I}\left[mgL_0 + \frac{8\pi l^2 EJ_z}{R^3(\pi^2 - 8)} \right], \quad (9)$$

where $I = mL_0^2$ is the pendulum inertia moment. A similar relation applies for circular frequency of concordant vibrations of two pendulums:

$$\omega_1^2 = \frac{mgL_0}{I} . \quad (10)$$

Having treated the relations (9), (10) and using vibration periods $T_1 = 2\pi/\omega_1$, $T_2 = 2\pi/\omega_2$ and well-known relation for area moment of inertia:

$$J_z = \frac{\pi d^4}{64} , \quad (11)$$

where d is wire diameter, we can obtain final relation for calculating elastic modulus of wire in the form of:

$$E = \frac{8(\pi^2 - 8)mgL_0R^3}{\pi^2 l^2 d^4}\left[\frac{T_1^2}{T_2^2} - 1 \right] . \quad (12)$$

Results of measurements

The measurements were carried out by means of connected pendulums as shown in diagram (Fig. 2). We have investigated the elastic properties of three materials – steel, aluminium and copper, all with the same geometric parameters (length and diameter). The values of elastic modulus E have been calculated from the formula (12), with common geometrical parameters used for all the samples (Table 1).

Table 1

Geometrical parameters of wire sample

L_0 [m]	m [kg]	d [mm]	R [m]	l [m]
0.84	0.87	1.4	0.16	0.25

Also the period of concordant vibrations was the same for all materials – T_1 = 1.040 s.

So, the only varying parameter had been the period of discordant vibrations T_2. The corresponding results for period of discordant vibrations are summarized in Table 2.

Table 2

Quantities measured for the determination of elastic modulus

Sample	T_2 [s]	E [GPa]	E_{tab} [GPa]
Steel	0.665	203.3	200–210
Aluminium	0.852	69.9	67–70
Copper	0.758	124.1	110–120

As we can see the obtained results are in good agreement with material-table values (last column); the differences represent no more as 5%.

Conclusion

The described equipment is simple and illustrative, completing the range of pendulum-based methods for the measurements of elasticity constants. Regarding 5% accuracy it ranges to the most accurate methods. It does non-require intricate measuring equipment and works without destruction, practically. Even extremely thin samples can be measured without a risk of damage or permanent deformation. The activity of pendulums is stable, the system phases do not „tune out" or dump even after several hundreds of oscillations. The method can be successfully used as a demonstration specimen in a university textbook (chapter „Vibrating Movements" or „Solids Physics"), or a task for laboratory exercises.

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Abstract

Classical reverse pendulums are currently used for measuring the gravity acceleration g, or – when pendulums bodies are connected by the spring – for demonstration of composition of parallel vibrations. In this paper we present the reversed pendulums in „non-traditional" position – as a device for measuring of elastic modulus of wire samples. The connection is realized by the measured wire sample with the circle shape.

Keywords: elastic modulus, connected reverse pendulums, thin wire samples.